DIVISION OF THE HUMANITIES AND SOCIAL SCIENCES

CALIFORNIA INSTITUTE OF TECHNOLOGY

PASADENA, CALIFORNIA 91125

AN INFORMATION-THEORETIC APPROACH TO JOB QUITS*

Louis L. Wilde

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ABSTRACT

This study analyzes the existence of quits as part of optimal job-shopping strategies by imperfectly informed workers. Formally this is modeled as a sequential decision problem in which jobs are assumed to be described by more than a single parameter. These multiple characteristics vary in their respective degrees of observability. Along with a characterization of the optimal strategy and a proof of the existence of a positive quit rate, comparative statics results are obtained.

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I. INTRODUCTION

Models of job search under imperfect information have become a commonplace in the Economics Literature.[1] The fundamental problem is the following. An unemployed worker desires to work but he does not know the characteristics of jobs at different firms. He can, however, observe various aspects of jobs by investing resources in "search" activity. Typical assumptions used in modeling this process are that jobs are identical except for wage and that the worker desires to maximize expected income net of search costs. In this case, assuming an infinite horizon and no discounting, it is easy to show that the optimal policy is to search sequentially and set a reservation wage, ξ, according to

$$c = \int_{\xi}^{\infty} (w - \xi) f(w) dw, \tag{1}$$

where c is the unit cost of search and f is the market distribution of wage offers. The decision rule is to accept the first offer which exceeds ξ. Even in this simple setting, a number of useful predictions stem from (1).[2] In particular, ξ is inversely related to c so that workers with higher search costs set lower reservation wages, expect to search less to find an acceptable job, and expect lower net incomes.

This primitive search model has been extended and generalized in many directions. But two issues are of interest here. The first concerns modifications in the optimal search strategy introduced when jobs are described by more than a single characteristic, the second concerns the role of search theory in explaining the existence and nature of job quits as an equilibrium phenomenon.

The first issue has been studied by Rosen [14], for example, in the context of tied-sales in product markets. His results suggest that as long as all the relevant characteristics of a job can be observed prior to any on-the-job experience, then the nature of the search strategy is not altered in any fundamental way -- the reservation wage is replaced by a reservation frontier which defines the trade-off between minimally acceptable bundles of characteristics. However, this conclusion is no longer valid if some of the characteristics of jobs are "specific" in the sense that they cannot be observed without actually working on the job.

A second problem with the basic search model defined above is that it doesn't allow for the possibility of quits. Clearly, in that world, if a searching worker finds a job offer which exceeds his reservation wage, then he accepts it and never quits. The implicit assumption in most search models is that the cost of searching while employed is so high as to preclude any on-the-job search. Burdett [3] provides a recent exception to this rule.[3]

Burdett allows the worker several possible modes of activity; he can search while unemployed, search and work, or work and not search at all (of course, the worker can also not work and not search, but this is uninteresting). If the cost of search while working is not too high relative to the cost of search while unemployed, then it pays the worker to set two reservation wages, $\xi_1 < \xi_2$, such that jobs offering wages less than ξ_1 are rejected, jobs with wages between ξ_1 and ξ_2 are accepted with search continuing, and jobs offering wages above ξ_2 are accepted with search activity ceasing altogether.

Burdett's formulation of the search problem introduces the possibility of quits in a framework of imperfect information and, assuming workers have finite lifetimes, his results are theoretically consistent with the empirical observation that quit rates are inversely related to both job tenure and age. These kinds of results are important because they provide a foundation for the existence of quits as an equilibrium phenomenon,[4] and highlight the importance of imperfect information in explaining labor market activity.

An alternative to Burdett's model is to assume that some characteristics of jobs are specific; that is, they cannot be observed without actually working the job.[5] The process of learning about such characteristics will be called evaluation. Characteristics which can be observed directly (upon paying some search cost) will be called general characteristics and those which require on-the-job experience will be called specific characteristics.[6] In light of Rosen [14], a single general characteristic will suffice. Suppose there are n specific characteristics, $y_1,\ldots,y_n$, and suppose further that these must be evaluated sequentially. If the sequence of evaluation is optimal in an appropriate sense, (roughly, the characteristics are observed sequentially in order of importance)[7] then quit rates decline with job tenure since good observations on early y_i's (where a good observation is one that does <u>not</u> induce the worker to quit) will reduce the likelihood that subsequent observations of other specific characteristics will result in a quit. This reduction in the probability of quitting reflects a true accumulation of knowledge.[8]

The next section of this paper presents a formal model of search and evaluation where there is just one general characteristic, the wage rate (w), and one specific characteristic (y) associated with jobs. It is established first that the worker's problem is well-defined, that an optimal policy exists and is unique, and that it is described by a sequential strategy. Next it is shown that under reasonable conditions the optimal strategy is to set a reservation wage, ξ, and search sequentially until a wage which beats the reservation wage is located. After working the job for one period, the specific characteristic associated with the (already known) wage is observed and the quit decision is made. Associated with the specific characteristic is a reservation function, $y^*(w)$, which defines acceptable ranges of y and ranges of y for which it is optimal to quit and search again.

In section 3 it is established that as long as the reservation wage is positive then there is a positive probability of quitting after taking a job. Section 4 considers changes in the optimal strategy when the period of specificity (the time it takes to evaluate the specific characteristic) changes. A concluding section summarizes the results and suggests some problems for further research.

Although this paper presents a theory of the existence of quits which is consistent (at least) with casual observation, it should be emphasized that it refutes neither the theoretical influence of human capital on the quit rate, nor other theories of the existence of quits. For example, it is clear that on-the-job search is important in explaining some quits, regardless of the existence of specific characteristics; Matilla [7] has estimated that approximately 60 percent of those who quit already have jobs lined up. But the decision to initiate search while employed may arise from several sources; it may be part of an optimal search strategy (see Burdett [3] or Lippman and McCall [5]), it may reflect a career switch point between jobs offering different rates of accumulation of human capital (see Rosen [13]), or it may be due to a poor outcome in the evaluation of some specific characteristic (see Pencavel [11]). While this paper considers only search while unemployed, relatively straightforward extensions would incorporate cases where employed workers initiate on-the-job search as a result of poor observations of some specific characteristic.

II. A MODEL OF SEARCH AND EVALUATION

2.1 Basic Assumptions

Underlying this model is the notion that unemployed workers seek jobs but are imperfectly informed as to the characteristics of available jobs. Jobs vary with respect to both wage and some nonwage characteristic. While ignorant of specific packages of the wage and nonwage characteristic offered by specific firms, workers can invest in information. The formal structure of this process is defined by the following six assumptions.

A1) Each job is described by a pair (w,y) where w is wage and y is some nonwage characteristic which cannot be observed without working on-the-job for one period; w is observed by direct sampling.

A2) The market distribution of W and Y is defined by a joint p.d.f. $\phi(w,y)$. Assume ϕ is positive on R_+^2. Let $f(w)$ and $g(y)$ be the marginal p.d.f. of W and the marginal p.d.f. of Y, respectively, and define $g(y|w)$ as the conditional p.d.f. of Y given $W=w$.

A3) Let $u(w,y)$ be the per period utility of working a job which pays wage w and possesses nonwage characteristic y. Assume u is differentiable and nondecreasing in each argument. Assume further that u is bounded.

A4) The cost of drawing an observation from ϕ is $c \geq 0$. Only one firm can be sampled per period, and the worker cannot sample and work simultaneously. Assume c is measured in the same units as $u(w,y)$ with $E[u(W,Y)] > c$.

A5) The worker's objective is to maximize his discounted stream of utility, net of search costs.

A6) The worker knows ϕ with certainty, sampling is without recall, the horizon is infinite, and the discount rate is β, $0 < \beta < 1$.

2.2 The Searcher's Problem

Given the six assumptions of 2.1, it is clear that the form of the optimal strategy will be sequential. The searching worker's problem is therefore the following. He knows ϕ. By paying c he can draw an observation, say (w_o,y_o), but he only sees w_o. Information concerning y_o is limited to knowledge of the conditional distribution of Y given w_o. After observing w_o, the worker can either accept the job or sample again; he cannot both accept the job and draw another observation. If he accepts the job he receives $u(w_o,y_o)$ for as many periods as he stays. But the worker will not discover that y_o is the true value of Y until he has worked for one period. If w_o is high, he may accept the job assuming y_o to be the average according to $g(y|w_o)$, only to discover by the end of the first period of employment that y_o is very low, for example. If it is low enough, the worker may quit the job and sample again. Consequently, for some values of W it may pay to take the job, knowing ahead of time that the probability of quitting is nonzero. The optimal policy is thus a kind of sequential-lexicographic process whereby the decision whether to take the job or not is based on critical values of W and the decision whether to stay or quit, obviously conditional on having accepted the job, is based on critical values of Y.

In the absence of any constraints on the distribution of W and Y there is no à priori relationship between an observed wage and the expected per period utility from accepting a job paying that wage. In fact, if high wages are strongly correlated with low nonwage characteristics, it might pay to reject high wage offers in favor of low wage offers.[9] In the body of this paper enough structure will be imposed on ϕ to guarantee that not only are high wages strictly preferred to low (prior to the observation of the nonwage characteristic) but, in addition, the reservation wage will be unique. In this case the optimal strategy is characterized at each stage of the process by a number, ξ, and a function, $y^*(w)$, as follows.

a) if $w < \xi$, then sample again

b) if $w \geq \xi$, then take the job for one period and (2)

(i) quit and sample again if $y < y^*(w)$

(ii) keep the job if $y \geq y^*(w)$.

2.3 The Functional Equation

The searcher's objective is to maximize his discounted stream of utility net of search costs. Define $v(w)$ as the expected value of the discounted stream of utility, net of search costs, generated by pursuing an optimal policy from the current period on when w has just been observed. Let V be the expected value of $v(w)$ taken with respect to the marginal distribution of w. Then

$$v(w) = -c + \beta\max\{V\,;\int_0^\infty [u(w,y) + \beta\max\{\frac{u(w,y)}{1-\beta}, V\}]g(y|w)dy\}$$

$$= -c + \beta\max\{V\,;\,E[u(w,Y)] + \beta[VG(y^*(w)|w) + \int_{y^*(w)}^\infty \frac{u(w,y)}{1-\beta}g(y|w)dy]\} \tag{3}$$

where $G(y|w)$ is the c.d.f. associated with $g(y|w)$, $y^*(w)$ is defined by

$$\frac{u(w,y^*(w))}{1-\beta} \equiv V,^{10} \tag{4}$$

and

$$V = \int_0^\infty v(w)f(w)dw. \tag{5}$$

The decision process formalized in (3) is illustrated in figure 1. Note that $y^*(w)$ is that value of the nonwage characteristic which leaves the worker indifferent between keeping a job paying wage w and quitting to search. As a matter of notational convenience, define the expected utility of taking a job at wage w to be

$$T(w) \equiv E[u(w,Y)] + \beta[V\,G(y^*(w)|w) + \int_{y^*(w)}^{\infty} \frac{u(w,y)}{1-\beta} g(y|w)dy]. \tag{6}$$

The value of a job offering wage w is the sum of the expected utility this period plus the discounted expected utility from the next period on into the future. This latter quantity is equal to the expected value of search times the probability of quitting plus the expected value of keeping the job times the probability of not quitting. The effect of assuming y to be a specific characteristic is reflected in the second term in (6). That is, the expected value of keeping the job conditional on $y \geq y^*(w)$ is not v(w); there is learning which is specific in the sense that it is lost when the worker quits to search.

The immediate problem is to show (3) has a solution. This result is straightforward and follows from assumptions (A1) through (A6).

Theorem 1: There exists a unique, continuous, bounded solution to the functional equation (3).

Proof: The proof is presented in Appendix A.

Given a unique solution to (3), V can be taken as a constant. The reservation wage, ξ, is defined by

$$V = T(\xi) = E[u(\xi,Y)] + \beta\{V\,G(y^*(\xi)|\xi) + \int_{y^*(\xi)}^{\infty} \frac{u(\xi,y)}{1-\beta} g(y|\xi)dy\}. \tag{7}$$

As it stands, there is no guarantee that ξ is unique. There may be intervals of acceptable wages separated by intervals of unacceptable wages, or it may be that all wages or no wages are acceptable. The next lemma and theorem establish conditions under which (1) ξ is unique and (2) high wage offers are preferred systematically to low wage offers. The approach is to find a sufficient condition for $\partial T(w)/\partial w > 0$. In this case either all wage offers are acceptable $(\xi = 0)$ or there is a unique $\xi > 0$ such that $w < \xi$ is rejected but $w \geq \xi$ is accepted. The sufficient condition for $\partial T(w)/\partial w > 0$ puts restrictions on the conditional distributions $g(\cdot|w)$ and implicitly on ϕ.

Consider an unemployed worker who draws an observation of w_o. He is concerned with the unknown characteristic associated with w_o, but only to the extent that it effects the total utility of the job. That is, he is not concerned with the conditional distribution $g(\cdot|w_o)$ per se. Rather, his decision whether to accept w_o or to reject it is based upon the conditional distribution of utility which is induced by $g(\cdot|w_o)$ through $u(w_o,y)$. Define a random variable Z as the per period utility of a job paying some known wage and possessing unknown characteristic y. Let $\Psi_w(z)$ be the c.d.f. of Z when wage w has been observed. Then a sufficient condition for $\partial T(w)/\partial w > 0$ is that $\Psi_w(z)$ be stochastically increasing in w. Thus:

A7) $\partial \Psi_w(z)/\partial w < 0$ for all $w \geq 0$.

It is easy to see that (A7) generates a kind of monotonicity with respect to a worker's evaluation of wage offers. The only way a high wage offer could be rejected in favor of a lower wage offer is if the worker believes that the high wage is associated with such a low value of Y that the net utility of the high wage job is less than that of the low wage job. Assumption (A7) rules out such possibilities.[11] The next lemma formalizes this argument.

Lemma 1: Under assumptions (A1) through (A7), $\partial T(w)/\partial w > 0$ for all $w \geq 0$.

Proof: From (6), integrating T(w) by parts and differentiating with respect to w yields

$$\partial T(w)/\partial w = -\int_{-\infty}^{z(w)} \partial \Psi_w(z)/\partial w \, dz - \left(\frac{\beta}{1-\beta}\right)\int_{V(1-\beta)}^{z(w)} \partial \Psi_w(z)/\partial w \, dz\,, \qquad (8)$$

where $z(w) = \lim_{y\to\infty} u(w,y)$. Thus $\partial \Psi_w(z)/\partial w < 0$ implies $\partial T(w)/\partial w > 0$. q.e.d.

Theorem 2: Under assumptions (A1) through (A7), either $T(w) \geq V$ for all $w \geq 0$ in which case $\xi = 0$, or there exists a unique $\xi > 0$ such that $T(\xi) = V$. Moreover, $y^*(w)$ is decreasing in w.

Proof: First note that $\lim_{w\to\infty} T(w) > V$ since otherwise $v(w) = -c + \beta V$ for all $w \geq 0$. In this case $V = -\frac{c}{1-\beta}$ which contradicts the assumption that $E[u(W,Y)] > c$. Since $T(\cdot)$ is

increasing (see Lemma 1), the first part of the theorem follows by rewriting (3) as $v(w) = -c + \beta\max\{V, T(w)\}$. Because $\partial u(w,y)/\partial y > 0$ and $\phi > 0$, inspection of (4) reveals that $dy^*(w)/dw < 0$. q.e.d.

Since $\partial T(w)/\partial w > 0$, it must be that wage offers below ξ are rejected and wage offers above ξ are accepted for at least one period. Thus, to the extent that optimal behavior after accepting any given job is buried in T(w), the optimal policy in this problem is analogous to the optimal policy in a simple search model. The primary difference is the additional reservation function, $y^*(w)$, associated with the nonwage characteristic. Figure 2 illustrates ξ and $y^*(w)$.

2.4 The Optimal Strategy

Using (6), the functional equation (3) can be written

$$v(w) = -c + \beta\max\{T(w), V\}$$

Noting that $w < \xi$ implies $T(w) < V$ and $w \geq \xi$ implies $T(w) \geq V$, and taking the expectation of v(w) with respect to the marginal p.d.f. of w, gives

$$V = -c + \beta\{V F(\xi) + \int_{\xi}^{\infty} T(w) f(w) dw\}, \qquad (10)$$

where F is the c.d.f. associated with f. Rearranging (10) and using the relations $T(\xi) = V$ and $u(\xi, y^*(\xi)) = V(1-\beta)$ yields

$$c + u(\xi, y^*(\xi)) = \beta\int_{\xi}^{\infty} [T(w) - T(\xi)] f(w) dw. \qquad (11)$$

While this expression is not independent of V, it describes the logic of the optimal strategy in familiar terms. The value of taking a job at w and ξ is given by $T(w)$ and $T(\xi)$ respectively. The marginal cost of a unit of search is the direct cost c plus the opportunity cost $u(\xi, y^*(\xi))$ of shifting back the entire stream of utility one period when ξ has been observed (note that the opportunity cost is <u>not</u> $E[u(\xi, Y)]$ but, rather, it is the value of working for one period when both margins of indifference, ξ and $y^*(\xi)$, are reached). The right-hand side of (11) is the discounted expected return to a unit of additional search when the current offer is ξ (and there is recall).

In the general model a key comparative statics result concerns changes in c. The result is not surprising, but it will be useful later for comparison purposes of comparison.

<u>Theorem 3</u>: $d\xi/dc < 0$.

<u>Proof</u>: By definition $V = T(\xi, V)$. Taking the total derivative,

$$\frac{d\xi}{dc} = \frac{\partial V}{\partial c}\left[1 - \frac{\partial T}{\partial c}\right] / \frac{\partial T}{\partial w} .$$

Now $\partial V/\partial c < 0$ and $\partial T/\partial V = \beta G(y^*(w)|w)$ since $\partial T/\partial w > 0$. This implies $d\xi/dc < 0$. q.e.d.

Of course, since an increase in c implies V falls, it must also be the case that as c rises, $y^*(w)$ falls for any given w; so investment in both search and evaluation fall as c rises. The probability that a given wage is accepted is $[1 - F(\xi)]$. The

probability that a given (w,y) pair is acceptable conditional on $w \geq \xi$ is

$$[1 - F(\xi)]^{-1} \int_{\xi}^{\infty} [1 - G(y^*(w)|w)] f(w) dw. \tag{12}$$

As c rises $[1 - F(\xi)]$ and $[1 - G(y^*(w)|w)]$ both increase. Thus, while it is true that ex post the quit rate falls, we cannot be sure that ex ante the expected probability of a quit falls as c rises.

III. THE QUIT RATE

The purpose of this section is to establish some relationships between acceptable wages and acceptable values of the nonwage characteristic. In particular, it will be shown that as long as the reservation wage is interior to the distribution of wage offers, then the probability of quitting is strictly positive.

<u>Theorem 4</u>: If $\xi > 0$, then $y^*(\xi) > 0$.

<u>Proof</u>: Assume the opposite; let $y^*(\xi) = 0$. Then by the definition of $y^*(\xi)$ given in (4) and footnote 10, we have

$$V \leq \frac{u(\xi, 0)}{1 - \beta}. \tag{13}$$

But $V = T(\xi)$, so when $y^*(\xi) = 0$,

$$V = T(\xi) = \frac{E[u(\xi, Y)]}{1 - \beta}. \tag{14}$$

Equations (13) and (14) imply $u(\xi, 0) \geq E[u(\xi, Y)]$, which is a contradiction since u is increasing and $g(y|\xi)$ is positive on

$[0,\infty)$. Therefore $y^*(\xi) > 0$. q.e.d.

This theorem establishes that the nonwage characteristic is never irrelevant when it pays to differentiate firms by wage. As long as ξ is strictly greater than 0 then at ξ, and in a neighborhood of it, it pays to reject some jobs on the basis of the nonwage characteristic. Thus after observing y, the probability of a quit $G(y^*(w)|w)$ is strictly positive for wages near the reservation wage. While this is a strong result (it proves the existence of quits as part of an optimal job-shopping strategy) it does not say that $y^*(w) > 0$ for all wages greater than ξ. For high enough wage offers it may pay to accept any level of y in which case the probability of a quite at that wage is zero.

It should also be pointed out that $\xi = 0$ does not imply $y^*(w) = 0$ for all $w \geq 0$. That is, even if all wage offers are acceptable, some levels of the nonwage characteristic may be rejected. To see this consider the following example. Assume $\xi = 0$ and $T(0) = V$. Then $y^*(0) > 0$. The argument proving this is similar to the proof of Theorem 4; by assuming $y^*(0) = 0$ one can reach a contradiction.

Finally, note that Theorem 4 does not rely on (A7). Even if there are multiple values of w such that $T(w) = V$, it remains true that at each of them (and in the proper neighborhood of each) the probability of a quit is nonzero.

IV. VARIABLE EVALUATION TIME

The model analyzed in this paper concerns investment in

two kinds of information, general and specific. The cost of investment in general information is c, the direct cost of search. The cost of investment in specific information is the opportunity cost of working for one period at a low $u(w,y)$ level. An increase in the cost of general information is simply an increase in c. An increase in the cost of specific information is equivalent to forcing the worker to work for more than one period before observing y. The purpose of this section is to incorporate such a modification into the basic model and then investigate the effects of changes in the cost of investment in specific information.

The modification is fairly straightforward. As before, the length of time it takes to generate an offer via search is one period, but now suppose it takes s of these periods to evaluate the unobservable characteristic y. In general, the worker would "learn" about the true value of Y during this initial stage of work (observing various random variables and forming expectations about their distributions), but we abstract from this process by fixing the evaluation period and assuming the worker receives $E[u(w,Y)]$ during each of the first s periods.[12] Hence the only formal change in the model is that upon taking a job at wage w the worker now expects to receive $E[u(w,Y)]$ for s periods, after which he makes a decision with respect to quitting or retaining the job.[13]

In the remainder of this section, the variables of the model will be indexed by time subscripts or superscripts to reflect the period of specificity. Assumptions (A1) through (A7) will again be assumed to hold where (A1) is modified to allow for evaluation periods greater than 1. Under these circumstances $y_s^*(w,V^s)$ is still defined by (4) and ξ_s is unique with wages below ξ_s rejected and wages greater than or equal to ξ_s accepted. The optimal return is now defined by

$$V^s(w) = -c + \beta \max\{T_s(w), V^s\}, \tag{15}$$

where in this case the value of taking a job at wage w is

$$T_s(w) = E[u(w,Y)]B(s) + \beta^s\{V^s G[y_s^*(w)|w] + \int_{y_s^*(w)}^{\infty} \frac{u(w,y)}{1-\beta} g(y|w)dy\} \tag{16}$$

and $B(s) = \sum_{i=1}^{s} \beta^{i-1}$. The first step in analyzing the effects of an increase in s is to show $V^s > V^{s+1}$. That is, as the period of specificity increases, the expected utility of optimal search falls.

Lemma 2: For each $s \geq 1$, $V^s > V^{s+1}$.

Proof: Let ξ_{s+1} and $y_{s+1}^*(w)$ be optimal when evaluation takes $s+1$ periods and let V^{s+1} be the expected value of search when these strategies are used in the $(s+1)$-period problem. Consider the following new problem: Suppose that the first job which is accepted has a nonwage characteristic which can be evaluated in s periods but all subsequent jobs require $s+1$ periods to evaluate the nonwage characteristic. Thus if the first job which is accepted results in a quit, the value of optimal search from then on out is V^{s+1}. Define the value of taking a job under these circumstances as $\hat{T}_{s+1}(w,V^{s+1})$. Then

$$\hat{T}_{s+1}(w,V^{s+1}) = E[u(w,Y)]B(s) + \beta^s\{V^{s+1}G[y_{s+1}^*(w)|w] + \int_{y_{s+1}^*(w)}^{\infty} \frac{u(w,y)}{1-\beta} g(y|w)dy\}.$$

The definition of $T_{s+1}(w,V^{s+1})$ given in (16) and the definition of $y_{s+1}^*(w)$ yield

$$\hat{T}_{s+1}(w,V^{s+1}) - T_{s+1}(w,V^{s+1}) \geq 0. \tag{17}$$

Define $U^1(w)$ as the expected value of an observation of wage w under the above conditions. Then

$$U^1(w) = -c + \beta \max\{V^{s+1}, \hat{T}_{s+1}(w,V^{s+1})\}$$

Let $\overline{U}^1$ be the expected value of $U^1(w)$ with respect to the marginal p.d.f. of wages, f(w). Then (17) implies $\overline{U}^1 \geq V^{s+1}$. Repeat the above procedure using $\overline{U}^1$ as the value of search but maintaining use of the strategies ξ_{s+1} and $y_{s+1}^*(w)$. That is, define $\hat{T}_{s+1}(w,\overline{U}^1)$ as the value of taking a job at wage w when the first two jobs can be evaluated in s periods but all subsequent jobs take $s+1$ periods to evaluate. It is trivial that $\hat{T}_{s+1}(w,\overline{U}^1) \geq \hat{T}_{s+1}(w,V^{s+1})$. Next define $U^2(w)$ and $\overline{U}^2$ analogously to $U^1(w)$ and $\overline{U}^1$. Then $\overline{U}^2 \geq \overline{U}^1$. Repeating the process generates a sequence $\overline{U}^1 \leq \overline{U}^2 \leq \ldots \leq \overline{U}^n \leq \ldots$. The limit of this sequence represents the case in which all evaluations take precisely s periods and the search strategy is characterized by ξ_{s+1} and

$y^*_{s+1}(w)$. Clearly this leads to an expected utility of search which is less than V^s. Thus $\{\overline{U}^i\}_{i=1}^{\infty}$ is nondecreasing and bounded above by V^s. Define $U^* = \lim_{n\to\infty} \overline{U}^n$ then

$$V^{s+1} \leq \lim_{n\to\infty} \overline{U}^n = U^* \leq V^s .$$

Strict inequality derives from the uniqueness of ξ_s and ξ_{s+1}. q.e.d.

Lemma 3: For each $s \geq 1$, $\partial T_s(w)/\partial w$ is positive and increasing in s.

Proof: Integrating by parts in (16), using $\Psi_w(z)$ as defined in section 2.3, and differentiating,

$$\partial T_s(w)/\partial w = -\left(\frac{1}{1-\beta}\right)\int_{-\infty}^{\overline{z}(w)} \partial\Psi_w(z)/\partial w \; dz$$

$$- \left(\frac{\beta^s}{1-\beta}\right)\int_{-\infty}^{V(1-\beta)} \partial\Psi_w(z)/\partial w \; dz. \tag{18}$$

where again $\overline{z}(w) = \lim_{y\to\infty} u(w,y)$. Since $\partial\Psi_z(w)/\partial w < 0$, $\partial T_s(w)/\partial w > 0$. Moreover, as s increases, both β^s and $V^s(1-\beta)$ fall, implying the second term in (18) gets smaller. Hence $\partial T_s(w)/\partial w < \partial T_{s+1}(w)/\partial w$. q.e.d.

Armed with lemmas 2 and 3, we are prepared to show that the reservation wage rises with s.

Theorem 5: For each $s \geq 1$, $\xi_{s+1} \geq \xi_s$ with strict inequality if $\xi_s > 0$.

Proof: Assume the opposite; that is, let $\xi_s > \xi_{s+1}$. In this case it must be that

$$T_s(w) - T_{s+1}(w) < V^s - V^{s+1} \tag{19}$$

for all $w \geq \xi_{s+1}$. This follows from the fact that when $\xi_s > \xi_{s+1}$ then $T_s(w) - T_{s+1}(w)$ is maximal at ξ_{s+1} over $w \geq \xi_{s+1}$. But $\xi_s \geq \xi_{s+1}$ implies this is less than or equal to $V^s - V^{s+1}$. With $\partial T_t(w)/\partial w$ increasing in t, the difference $T_s(w) - T_{s+1}(w)$ can only get smaller than $V^s - V^{s+1}$ as w increases above ξ_{s+1}.

Equation (19) implies $T_s(w) - V^s < T_{s+1}(w) - V^{s+1}$ for all $w \geq \xi_{s+1}$. Using (15) as in section 2.4 generates

$$c + V^t(1-\beta) = \beta\int_{\xi_t}^{\infty} [T_t(w) - V^t]f(w)dw. \tag{20}$$

Consider what happens in (20) as t goes from s to s + 1. V^t falls but $[T_t(w) - V^t]$ rises. Thus ξ_t must rise unless $\xi_s = 0 = \xi_{s+1}$, implying $\xi_{s+1} \geq \xi_s$. This contradicts the original assumption that $\xi_{s+1} < \xi_s$. Thus it must be that $\xi_{s+1} \geq \xi_s$. The strict inequality obviously holds when $\xi_s > 0$. q.e.d.

Theorem 6: For all $w \geq 0$, $y^*_s(w) \geq y^*_{s+1}(w)$ with strict inequality if $y^*_s(w) > 0$.

Proof: By definition $\dfrac{u(w,y^*_t(w))}{1-\beta} = V^t$. But V^t falls as t rises. With w fixed this implies $y^*_t(w)$ is falling as t rises. q.e.d.

The implication of Theorems 5 and 6 is that as s rises (the cost of evaluation increases) there is a shift away from evaluation toward search. Moreover, there is an unambiguous fall in the *ex ante* expected probability of a quit when s increases. This is in contrast to the case when c increases. There, while both search and evaluation decrease as w^* and $y^*(w)$ fall,[14] we cannot guarantee a fall in the *ex ante* expected quit rate a c rises.

V. SUMMARY AND CONCLUSION

The analysis in this paper has uncovered two strong results. The first is that while an increase in direct search costs decreases both search and evaluation, an increase in the costs of evaluation (a rise in s) causes a shift away from evaluation and towards search. The second strong result is that as long as wage offers matter (i.e. as long as there exists at least one ξ interior to the distribution of potential wage offers), then the quit rate is strictly positive (in a probabilistic sense).

These results forcefully illustrated the importance of distinguishing between general and specific information. In future research, this distinction will be applied to consumer behavior in product markets as well as other problems in labor economics. For example, the same sort of analysis can be used to explain firm's firing behavior; the initially unobserved characteristics of employees also become known after a certain period of employment.

APPENDIX

This appendix outlines a proof of theorem 1.

Theorem 1: There exists a unique, continuous, bounded solution to the functional equation (3).

Proof: Since $u(w,y)$ is continuous and bounded on R_+^2 and β satisfies $0 < \beta < 1$, a straightforward application of Denardo [4] suffices to establish theorem 1.

A more traditional approach is to define a sequence of truncated problems by the functional equations

$$v^N(w) = -c + \beta \max\{V^{N-1}; \int_0^\infty [u(w,y) + \beta \max\{u(w,y)B(N-1), V^{N-2}\} g(y|w)dy\},$$

where $B(N-1) = \sum_{n=1}^{N-1} \beta^{n-1}$, V^N is the expected value of $v^N(w)$ according to $f(w)$, and $N \geq 2$. Otherwise $v^1(w) = -c$ and $v^0(w) = 0$. Standard arguments show that there exists $(\cdot)$ defined on $[0,\infty)$ such that $\{v^N\}$ converges uniformly to v, v is the unique solution to (3), and is continuous and bounded on $[0,\infty)$. q.e.d.

FOOTNOTES

1. The literature on the economics of information, in general, and on job search, in particular, has become enormous. A recent survey of the latter is provided by Lippman and McCall [5].

2. This sequential version of the search problem is analyzed in McCall [8]. A concise summary of its implications is provided in Rothschild [16]. Although Rothschild's discussion is set in the product market, his model is easily translated into the labor market.

3. For a similar approach to job quits, see Lippman and McCall [5, pp. 179-181]. Also, since the completion of this work, I have become aware of two related papers. The first is a comment by George Borjas and Matthew Goldberg [2]. They attempt an extension of the basic search model which allows jobs to be described by multiple characteristics, some of which cannot be observed until after the job is taken. However, they do not integrate the possibility of quits into their model. Naturally in this case, given suitable assumptions regarding the market distribution of characteristics, the fundamental aspects of the simple search model are not altered. The second paper is by Dale Mortensen [9]. He models worker turnover based on stochastic learning of job characteristics, but does not distinguish between specific and general information or explicitly incorporate search into his model.

4. There are several explanations for the existence of quits. In the classical competitive model, voluntary separations are viewed as a response to disequilibrium states of net advantage across jobs. In equilibrium, the principle of equalizing differences holds that net advantage across all jobs will be the same, so that no worker ever has an incentive to quit.

Human capital theory, as expounded by Becker [1], does not explain the existence of quits, but it does suggest that quit rates should be inversely related to age and job tenure. For an application of human capital theory which does explain the existence of some quits as part of an optimal worker strategy see Rosen [13].

Burdett's model provides an alternative to these explanations based upon the concept of on-the-job search. As another information-theoretic alternative, the current analysis formalizes Pencavel's [11] observation that "the taking on of a job for a trial period may be the optimum method for an individual to discover whether that employment suits him."

5. For a more complete discussion of specific and general information see Wilde [17]. Note, however, that the definition of specific characteristic precludes the possibility of a firm informing potential employees of the exact nature of a job's nonwage characteristics before the worker accepts employment.

6. The term "evaluation" which is used herein to describe the process of learning about specific characteristics is borrowed from MacQueen [6]. MacQueen's model is not set in the labor market and while his "possibilities" are described by multiple characteristics, each of the characteristics can be observed by paying a search cost. The connection is that if the outcome of the first characteristic observed is poor enough, the searcher might choose not to observe the second characteristic at all, preferring to draw a new "possibility" for evaluation. But MacQueen, like Borjas and Goldberg, does not incorporate "turnover" into his model.

Also, the distinction between specific and general information is similar to the distinction Nelson [10] made between "search" goods and "experience" goods. However, Nelson's definitions are too strong. He defines search goods as goods for which only general characteristics matter and experience goods are defined as those for which only specific characteristics matter. As the model in section 2 will suggest, these definitions are polar cases and Nelson's analysis can be extended to intermediate examples.

7. It turns out that if the order of evaluation of specific characteristics is fixed exogenously, then quit rates need not decline with job tenure. This is because the worker may be forced to evaluate

irrelevant characteristics early in his work experience. If the worker is allowed to set the order of evaluation as part of an optimal strategy, then one would expect quit rates to be inversely related to job tenure.

8. This notion is discussed in Pencavel [11] who quotes Reynolds [12]: "Voluntary mobility is essentially a form of job-shopping by workers . . . workers have great difficulty in judging the attractiveness of a job by talking it over in the company's employment office. The only way to judge it accurately is to work on it for awhile. After a few weeks or months of work, one can tell whether the job is worth keeping. This explains why quits are most frequent during the first few months of service and diminish rapidly after that point."

9. This model is clearly a "partial partial-equilibrium" model, to use Rothschild's term. As such it must suffer the same criticism Rothschild [15] leveled against early models of job search: the model takes the distribution ϕ as given.

10. It is possible that for some w, $\frac{u(w,0)}{1-\beta} > V$. In this case we define $y^*(w) = 0$. Hence $y^*(w) = 0$ implies $\frac{u(w,y^*(w))}{1-\beta} \geq V$.

11. Assumption 7 is satisfied, for example, when W and Y are independent. It is also satisfied when the conditional distributions $g(\cdot|w)$ are stochastically increasing in w. However, this last condition implies that higher wages are on average associated with higher values of the nonwage characteristic. Not only is this strong, the classical concept of equalizing differences suggests that in equilibrium it probably isn't true (see footnote 9).

12. Alternatively, one might wish to think of s as the length of an initial contract (a "probationary" period). In this case Y might be observed after one period, but the quit decision postponed for s periods.

13. Of course it is only $E[u(w,Y)]$ which matters to the worker when he decides (before observing Y) whether or not to accept a job offering wage w. Moreover, what he actually receives during the first s periods of employment is qualitatively irrelevant to the quit decision. For example, one variation on this model is to assume W is the wage received during an "apprentice" phase and Y is the subsequent raise received at the end of s periods. Then $E[u(w,Y)] = u(w)$ during apprenticeship and $u(w,y) = u(w+y)$ after promotion.

14. Again, the spirit of Theorems 5 and 6 is preserved even if (A7) is dropped. If we define $A_s \equiv \{w | T_s(w) \geq V^s\}$ as the "acceptance set," then Theorem 5 becomes $\text{Prob}\{w \in A_s\} \leq \text{Prob}\{w \in A_{s+1}\}$ and Theorem 6 holds as stated.

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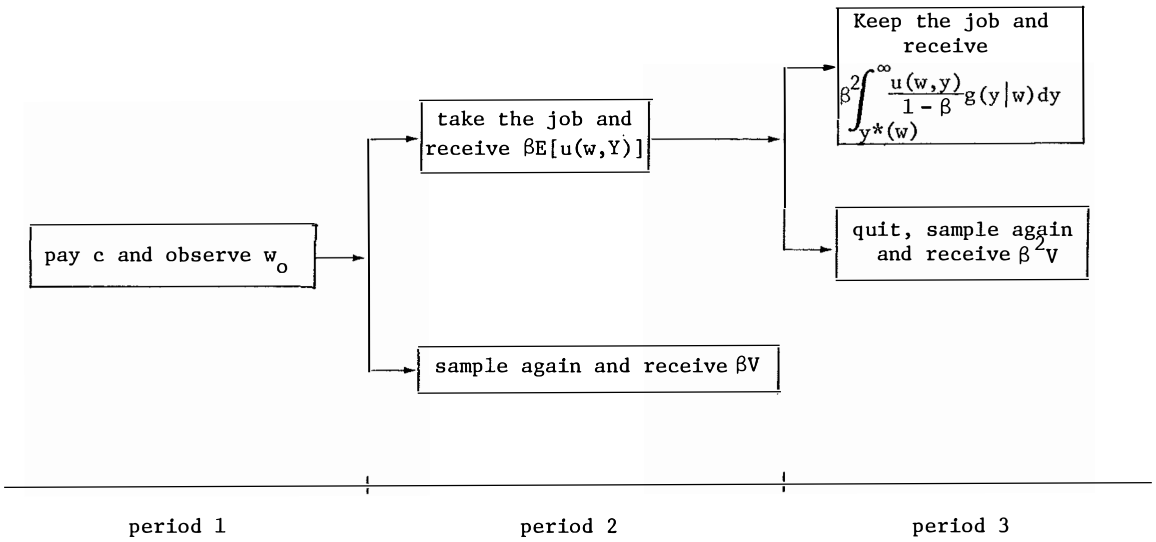

Figure 1: The Decision Process

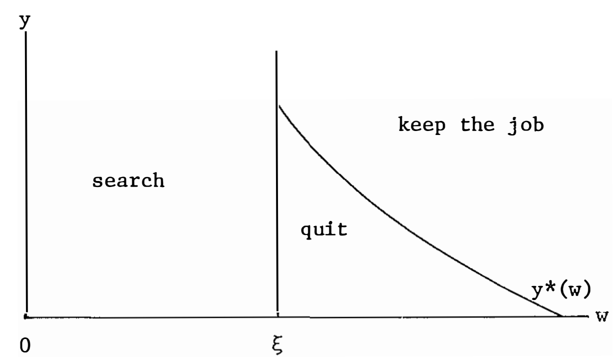

Figure 2; ξ and $y^*(w)$